Ethics in Mathematics Sample Exercise Questions

Exercise 1: Confronting Your Boss with Logic

Your boss has given you a task. The task is well within your technical capability, but you are not sure whether it would be legal or ethical. You feel uneasy, but your boss tells you: 'If you don't do it, then someone else will.'

Do you think that the boss' argument is cogent? Does that depend on who you are or what the project is? How would you answer your boss?

Exercise 2: Ethics of Crime Scene Investigation

A detective arrives at the scene of a crime at 5:00pm, finds a warm cup of tea, and measures its temperature at 40°C. By 5:30pm the tea's temperature has dropped to 30°C.

(a) The police approach you with this data and ask you when the tea was likely made. Briefly discuss any questions that you still need to ask the police officers and their potential ethical relevance. What are potential barriers of communication?

(b) The police are unable to provide you with more information, but ask you to give an estimate based on idealised conditions and a constant room temperature of 20°C. Giving all mathematical details and assumptions, use Newton's law of cooling to estimate when the tea was likely made. What error margins might your computation have, and what might the consequences of these be?

Exercise 3: Mathematics of Military Engagement

Consider two armies R (red) and B (blue). Let $m_R(t)$ and $m_B(t)$ be the number of soldiers of army R and B respectively.

(a) Assume that the losses of each army are proportional to the strength of the other army with proportionality constants a_R and a_B. Set up a system of differential equations that describe the strength of each army at time t.

(b) Derive Lanchester's square law

$$a_B \left(m_B^2(t) - m_B^2(0) \right) = a_R \left(m_R^2(t) - m_R^2(0) \right).$$

(c) How would you describe the proportionality constants a_R and a_B in everyday language? Would you be comfortable working on such problems in your career?

Exercise 4: Mathematical Communication

(a) Let $F : [a, b] \to \mathbb{R}$ be a smooth function with $F(a) < 0 < F(b)$, and $F'(x) > 0$. Then the equation $F(x) = 0$ has a unique solution $x = c$. Numerical approximations to c can be found using Newton–Raphson iteration, which converges quadratically.

State the formula for Newton–Raphson iteration and explain it graphically. Convey the above information to someone who has no post high school mathematical training.

(b) State the formulae for the Finite Difference methods (forward, backward and central) for the approximation of the first derivative of a function $F(x)$ at point a denoted by $F'(a)$ using a step size h, and explain it graphically. Convey this information to someone who has no post high school mathematical training.

Exercise 5: Uncertainty and Error Calculations

A projectile of mass m is fired from the (flat) ground with velocity $\mathbf{v}_0$, while a wind blows with constant velocity $\mathbf{u}$. The gravitational acceleration is $\mathbf{g}$ and the air exerts a drag force on the projectile equal to mk times the velocity of the projectile *relative to the wind.*

(a) Write down and solve the equation model to find the trajectory of the projectile (assuming the projectile starts at the origin $\mathbf{x_0} = \mathbf{0}$)

(b) Suppose that $\mathbf{v}_0$ is vertical and $\mathbf{u}$ is horizontal. Show that the travel time T of when the projectile hits the ground satisfies the implicit equation

$$1 - e^{-kT} = \frac{kT}{1 + \lambda},$$

where $\lambda = kv_0/g$, for $g = |\mathbf{g}|$, $v_0 = |\mathbf{v}_0|$. Verify that λ is nondimensional, and give a physical interpretation of λ.

(c) Although no analytic form is available for T, find an expression for the range R of the projectile (that is, the distance it travels before it lands) in terms of T and the other parameters.

(d) Real measurements can never be perfectly accurate or precise. Given that u is uncertain, estimate the uncertainty in T and R. *Hint:* Linearise the equations about the true values by writing $u = u_0 + \delta u$, where u_0 is the true value and δu is the uncertainty. Find an equation relating δT and δR to δu.

Extension: Now, estimate the error if v_0 is uncertain i.e. $v_0 = v_{00} + \delta v$ (you will need to use Taylor expansions to get the error estimate).

Why is it important to carry out uncertainty estimates in these calculations?

(e) This question has asked you to investigate *projectile motion*. Can you list any uses of projectile motion that do not relate to military applications?